Convert a circle into a cylinder, where the bottom equals the center of the circle. (This way e.g. we get the circle area, being exactly half of the cylinder surface, by polar coordinate integral).

All points at the bottom circumference are distinct, and remain distinct also if the operation is reversed, but at this infinitely small physical dimension (a point again) they are only distinct as different angles.

A black hole horizon can be the singularity. A non-local surface but still harboring distinct quanta.

The surface has to hold all quanta, will hence be proportional in size to their number (although we cannot tell any specific location for any quantum, this ‘surface’ is non-local and all quanta are entangled).

Why in particular now do we need $4\times (\text{Planck length})^2$ per quantum? We have the circle area, being $\pi r^2$, where $\pi$ represents the integral over the angle. Hence any elementary step on the edge of the circle is carrying the factor $\pi$. The surface of a sphere is $4\pi r^2$, hence the elementary surface area is 4 times as big. Hence four (Planck length)$^2$ instead of just one: It is simply the unit of the solid angle!

Let us recapitulate: We have shown a transformation from circle to point. Shrinking the circle transforms linear elements on it into angles. They are still parted even if we get to the point, but only as angles!

Hence, a point, or singularity, can still have many facets, or angles of observation.

In 2D, the elementary angles are unit 1, in 3D their elementary unit is 4. Therefore black hole entropy is in $4(\text{Planck length})^2$ units.
**Further explanation**

In a solid celestial body, space is compressed (geodesic lines become less dense and expand from the center) and a void is emerging inside. This driven to the extreme, all geodesic lines compress to a sphere which is nothing but a black hole horizon, and inside there is nothing (really nothing, not 'vacuum').

If we start the expansion from a single point (think of a crossing of geodesic lines), then the very point will expand and the BH horizon will exactly be the transformation of the singularity. Both will be identical. An infalling astronaut experiences himself falling into a point, while an external observer sees him being smeared over all of the horizon.

All the horizon is non local, all quanta in it must be entangled. The information the horizon can hold is given by the solid angle. A circle could hold as many qubits as it has Planck lengths, for the sphere nothing smaller than 4x Planck area can be distinguished.

Taking polar coordinates, the apparent contradiction between a point singularity and its property of holding many qubits is easily resolved, as the point has no extension but still many angles from which it can be observed. In polar coordinates, extension translates into angles - that's all it needs.

The illustration shows the expansion, centered at a coordinate knot (gray). An infinitesimal mesh element (red) is expanded in such a way that any direction from the origin is still distinguished at the horizon (blue circle, blue arrows).